PARAMETRIC DIFFERENCE-FREQUENCY GENERATION OF SOUND IN AIR

William Pierce Shealy

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

PARAMETRIC DIFFERENCE-FREQUENCY
GENERATION OF SOUND IN AIR

by

William Pierce Shealy

Thesis Advisor: A. I. Eller

December, 1972

Approved for public release; distribution unlimited.

Parametric Difference-Frequency Generation of Sound in Air

by

William Pierce Shealy
Lieutenant, United States Navy
B.S., United States Naval Academy, 1965

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING ACOUSTICS

from

NAVAL POSTGRADUATE SCHOOL

December, 1972

ABSTRACT

This study explores the feasibility of parametric difference-frequency generation in air through the interaction of two intense high-frequency sound waves. The sound pressure levels at various distances from the source are shown and the directivity of the difference-frequency is described.

An experimental apparatus utilizing a modified St. CLAIR resonator to generate the high-intensity primary signals is explained and some characteristics of the resonator are described.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

The interaction in air of two intense sound waves of coincident direction of propagation and different frequencies results in the formation of combination waves. This study deals with the formation of lower frequency combination wave, the difference wave. It is believed that the difference-frequency wave has greater engineering applications because of its frequency and its high directivity.

It is an accepted fact that the difference wave can be created in water through parametric amplification. However, there seems to be some question about the feasibility of this occurrence in air. This study attempts to determine if, in fact, the difference-frequency can be generated in air and, at the same time to describe an experimental apparatus which is capable of producing the necessary high-intensity primary sound fields.

The study was conducted with the primary frequencies in the near ultrasonic range. The availability of equipment (amplifiers, microphones, etc.) for operation in this range made it advantageous to operate there. Since the sound pressures of parametric difference-frequency grow with increasing difference this range gives a wide selection of possibilities for the difference-frequency itself.

II. GENERAL METHODS

In order to study the feasibility of difference-frequency generation by a parametric interaction of two signals it is necessary to generate the source frequencies by some means, and then to combine them in the desired medium. In general, two methods, two different approaches to the problem, were attempted. Each method offers its own advantages and at the same time poses some additional problems.

With the first method, problems of "mixing" in the electronics were avoided while it posed questions of geometric misalignment. The second method overcame any problems in the geometry of the source but required some filtering technique to eliminate difference-frequency generation in the electronics.

A. METHOD 1

This method utilized two oscillators, each set at one of the primary frequencies; two amplifiers; and two acoustic sources. In this way the two primary waves were electronically isolated and were mixed only in the air as acoustic waves (Fig. 1). However, the phenomenon of parametric amplication is extremely sensitive to the geometry of the region of interaction of the two primary waves, and using this method it was impossible to determine if a satisfactory region was produced.

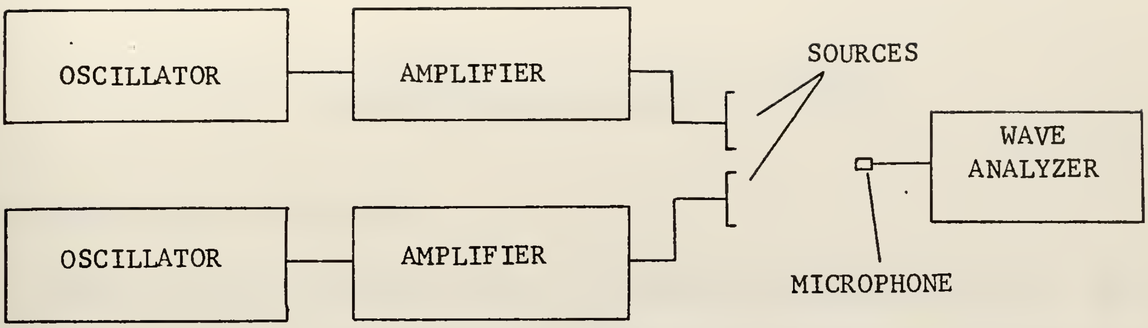

FIG. 1 METHOD 1

B. METHOD 2

With this method again the two oscillators were set at the primary frequencies and a bridge was used to combine them. A band-pass filter was used with a pass band about 2kHz below the lower primary to 2kHz above the upper primary. A good quality pre-amplifier and amplifier with minimum distortion levels was used to amplify the signals to high enough power to be used by the source. Since both primary acoustic waves come from the same source, geometric considerations for a region of overlapping primaries were minimized (Fig. 2). Because any acoustic source responds differently at different frequencies there are still some questions concerning the region of interaction but the parallax problem was eliminated.

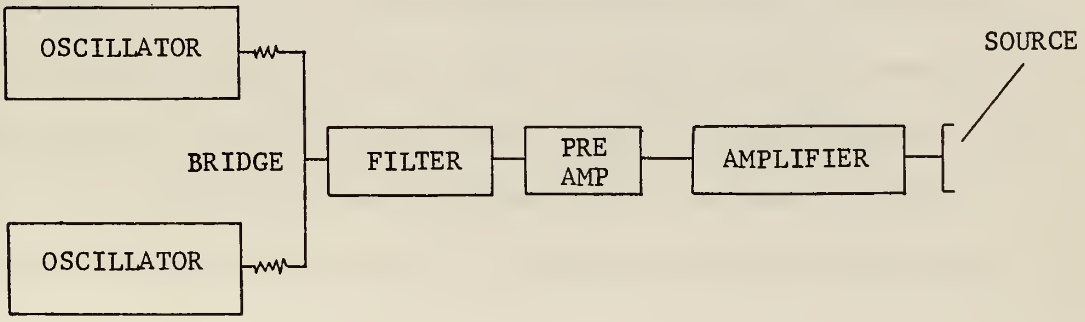

FIG. 2 METHOD 2

III. DESCRIPTION OF ELECTRONIC EQUIPMENT

A. SELECTION OF AMPLIFIERS

Several amplifiers were tested to determine distortion levels. Specifically, the type of distortion least desirable was the generation of a difference-frequency in the amplifier when two separate frequencies were the input. Harmonic distortion was a minor consideration. It was found that generation of a difference-frequency in the amplifier effectively masked any parametric generation in the medium. For that reason, it was necessary to eliminate it where possible or reduce it to the lowest levels. Figure 3 shows the output of a Bogen amplifier that was tested. The input consists of an 19.0kHz signal and a 21.5kHz signal of the same level. The level of the distortion component at the difference-frequency was unacceptable. This may be compared to Figure 4, which is a graph of the output of the Krohn-Hite amplifier that was used for the later data-collection phases of this study. The input to the amplifier for this graph was the same as before but the distortion was at an acceptable level.

The amplifier used in the data-collection phase was a Krohn-Hite, wideband, 50-watt amplifier, model DCA-50. It has an input impedance of 5000 ohms and a maximum output current (rms) of 550 milliamps. The 200-ohm output impedance of the amplifier may be matched to loads of 2, 8, 32, or 128 ohms with the use of the MT-56 matching transformer.

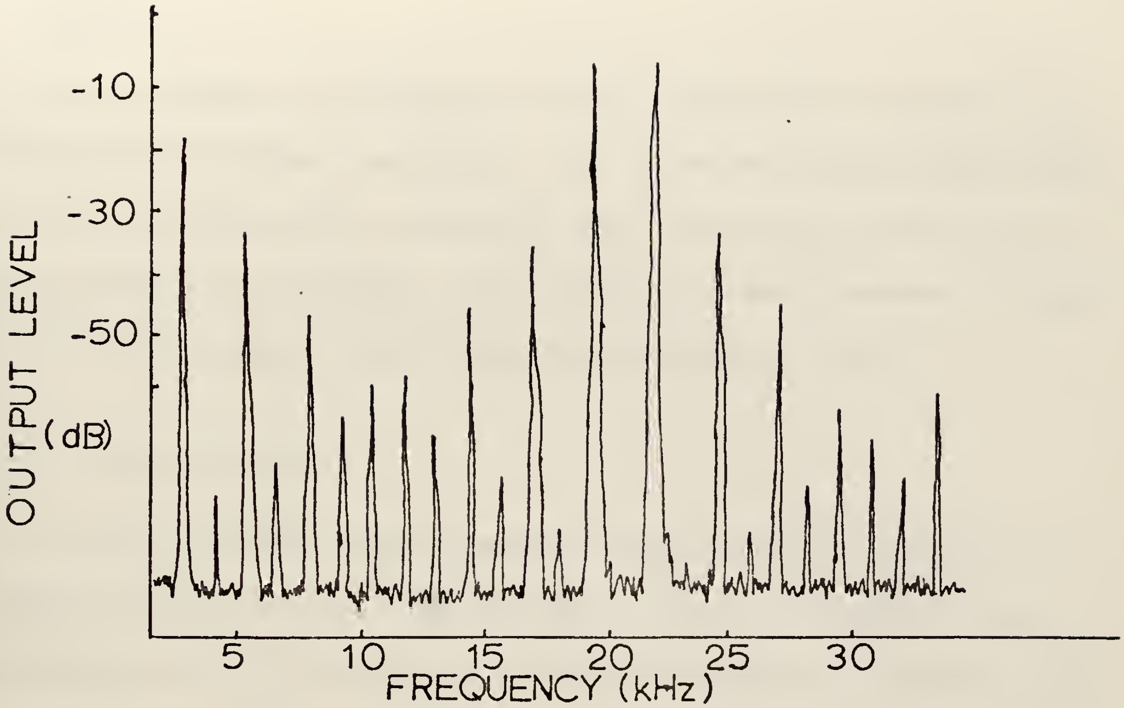

FIG. 3 OUTPUT VS FREQUENCY
for a Bogen amplifier with inputs of 19.0kHz and 21.5kHz.

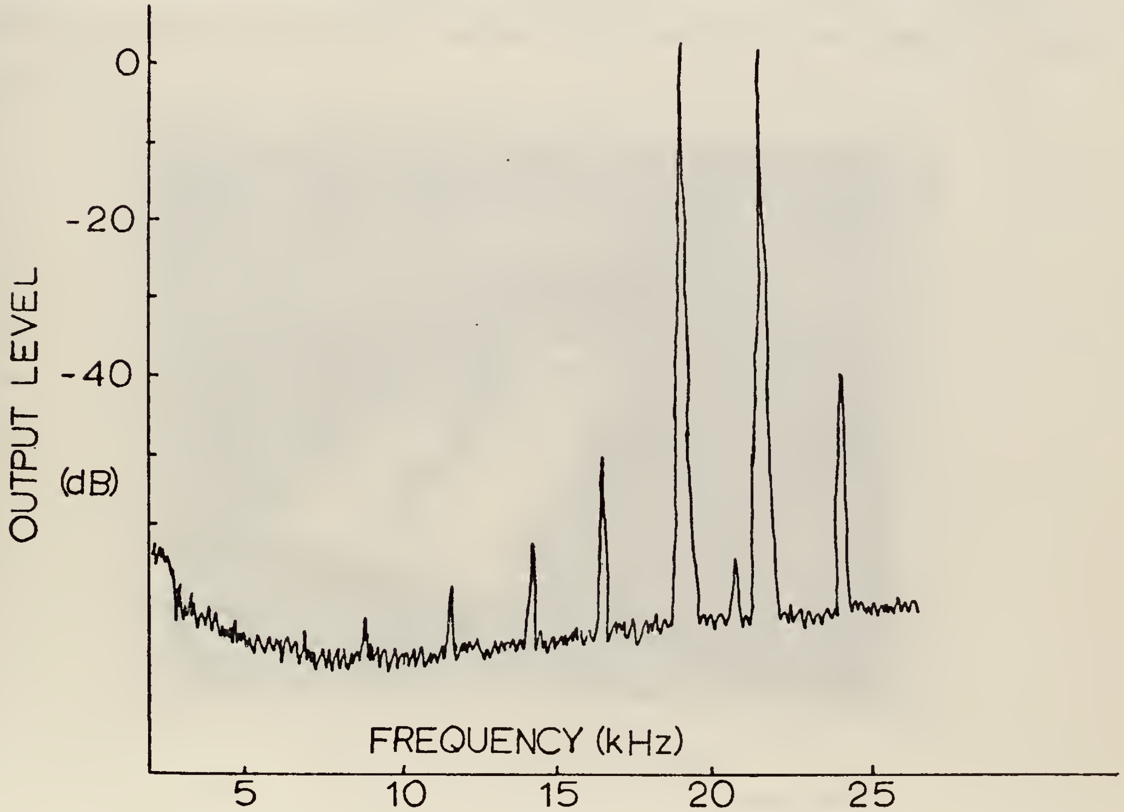

FIG. 4 OUTPUT VS FREQUENCY
for Krohn-Hite amplifier with inputs of 19.0kHz and 21.5kHz

B. SPEAKERS

Several attempts at difference-frequency generation were made using "off-the-shelf" speakers operating at the upper end of the audible range. The speakers used were two University T-202 "Sphericon" tweeters with an input impedance of eight ohms. The response of these speakers is minimal below 3kHz and maximal in the range between 15kHz and 20kHz.

C. ST. CLAIR RESONATOR

In order to produce primary acoustic signals of high intensity, a resonant vibrator was used. This vibrator operates according to the principle given by St. Clair [Ref. 1] and was designed to operate in the ultrasonic range. A similar device was used by Brinkmann in a study of parametric sound [Ref. 2]. It consists of a massive, slightly damped aluminum cylinder excited by a ceramic disk bonded to one face by a good quality epoxy (Fig. 5).

FIG. 5 ST. CLAIR RESONATOR
with a ceramic disk used as a driving unit

Several resonators were built varying in length from 10.0 cm to 11.3 cm with a diameter of 12.7 cm. Different sizes of barium titanate disks were bonded to the resonators in an attempt to achieve an optimum design. Because of the high-Q nature of the resonances of such a vibrator a stable oscillator is needed as a signal source. A General Radio frequency synthesizer was used for this purpose.

D. MICROPHONE AND WAVE ANALYZER

A condenser microphone made by Bruel and Kjaer was used as a measuring microphone. It consists of a microphone capsule of one-quarter inch diameter with a connection adapter and cathode follower. It possess a practically frequency independent sensitivity of -77 dB re 1 volt/.0002 μbar below 100kHz. Calibration was done using a Bruel and Kjaer pistonphone, model 4220. The sound pressure levels in the sound field were determined by a Hewlett-Packard wave analyzer, model HP 3590A, with a selectable bandwidth of $\pm$ 100 Hz. The experiments were performed in the anechoic chamber of the United States Naval Postgraduate School, which is equipped with a rotating platform apparatus for directivity determinations.

IV. CHARACTERISTICS OF RESONATORS

A. DRIVING UNIT

The St. Clair resonator described in Ref. 1 used an electromagnetic driver. However, in this study a cermaic disk was used in order to make it possible to move the resonator for directivity measurements. The ceramic makes the resonator a more or less portable sound source.

B. SPEED OF SOUND DISPERSION

The resonator was designed to be excited into longitudinal vibration by the ceramic disk. With a length of 11.22 cm and a diameter of 12.7 cm, the fundamental longitudinal resonance ($\lambda/2$ resonance) was computed to occur at 19.9kHz and the next longitudinal resonance ($2\lambda/2$ resonance) at 29.9kHz [Ref. 3]. These resonant frequencies are not harmonically related due to the dispersion of the speed of sound in cylindrical rods. The values of the speed of sound in solid cylinders of this nature are tabulated by Bancroft [Ref. 3] as a function of Poisson's ratio and the diameter to wavelength ratio, D/λ. For a bar of these dimensions the speed of sound the cylinder at the fundamental resonance is $.556c_o$ and for the next resonance is $.654c_o$ where c_o is the speed of sound for an infinitely long cylinder (5150 m/sec in aluminum). These resonances are in agreement with the experimentally ascertained resonant frequencies.

C. DISCUSSION OF RESPONSE AND SOURCE STRENGTH

Frequency response curves were run on this resonator configuration (Fig. 6a) and the presence of multiple resonances was note. Booker and Sagar [Ref. 4] described these additional resonances as doublets or multiplets. Figure 6a is the response of a vibrator of length, L = 11.22 cm, and diameter, D = 12.7 cm, with a ceramic disk of diamter, d = 6.3 cm, and thickness, t = 6 mm. Figure 6b shows the response of a vibrator of the same dimensions driven by a disk with d = 5.1 cm and t = 6 mm. The resonant frequencies are essentially the same, as expected. However, the relative strength of each resonance is different. After studying the response of several resonator configurations it was concluded that the difference of the relative strength of each resonance is greatly dependent on the quality of the epoxy bond between the ceramic and the aluminum. At the same time, the characteristic resonant frequencies of the ceramic itself have some effect on the response of the configuration. Figures 6a-6f show the response of various resonator configurations. Note the sharp peak of most of the resonances. The half-power point (down 3 dB) of the resonance occurs at $\pm$ 10Hz from the peak.

The maximum obtainable sound pressure levels (SPL) using the St. Clair resonator exceed 140 dB re .0002 μbar. A typical source level (SPL at 1 meter from the source) was 125 dB for the strongest resonance. Driving voltages of 40-80 volts were necessary to achieve such a response.

D. DIRECTIVITY AND MODE DEPENDENCE

The directivity of the longitudinal mode resonances, that is, those resonances which occur at or very near the calculated longitudinal waves resonance, are characterized by a single strong major lobe of about 10 degrees in width at the 3 dB point (Fig. 7). This type lobe structure

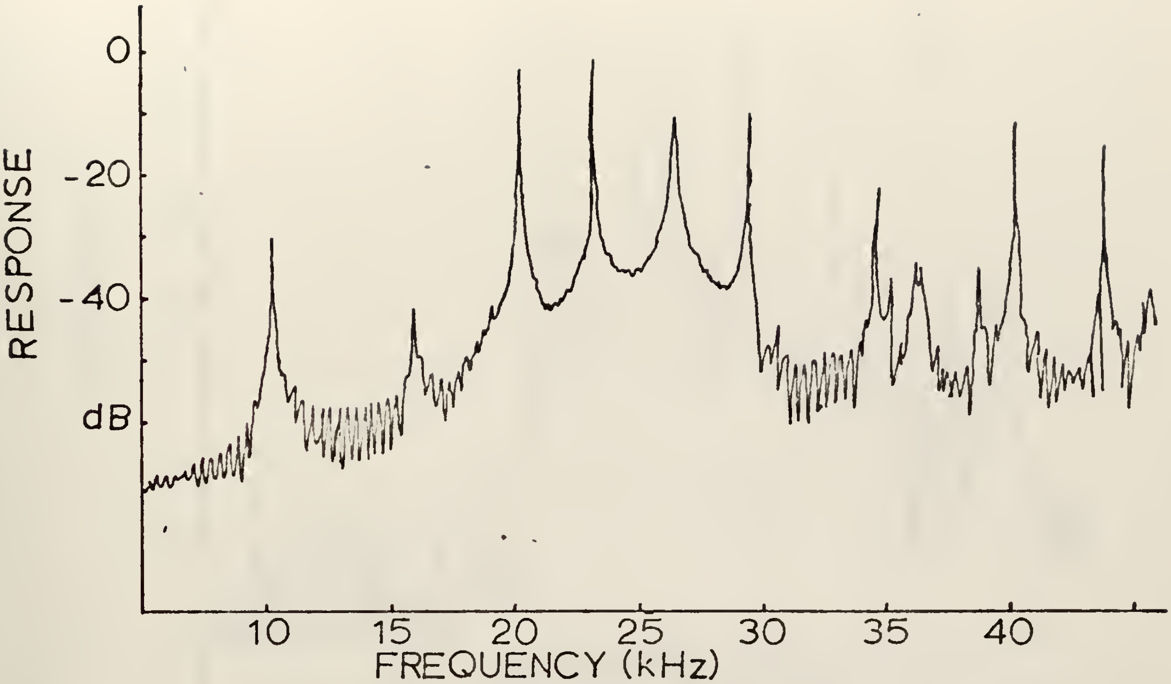

FIG. 6a OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR
With l = 11.22cm, D = 12.7cm, d = 6.3cm, t = 6mm

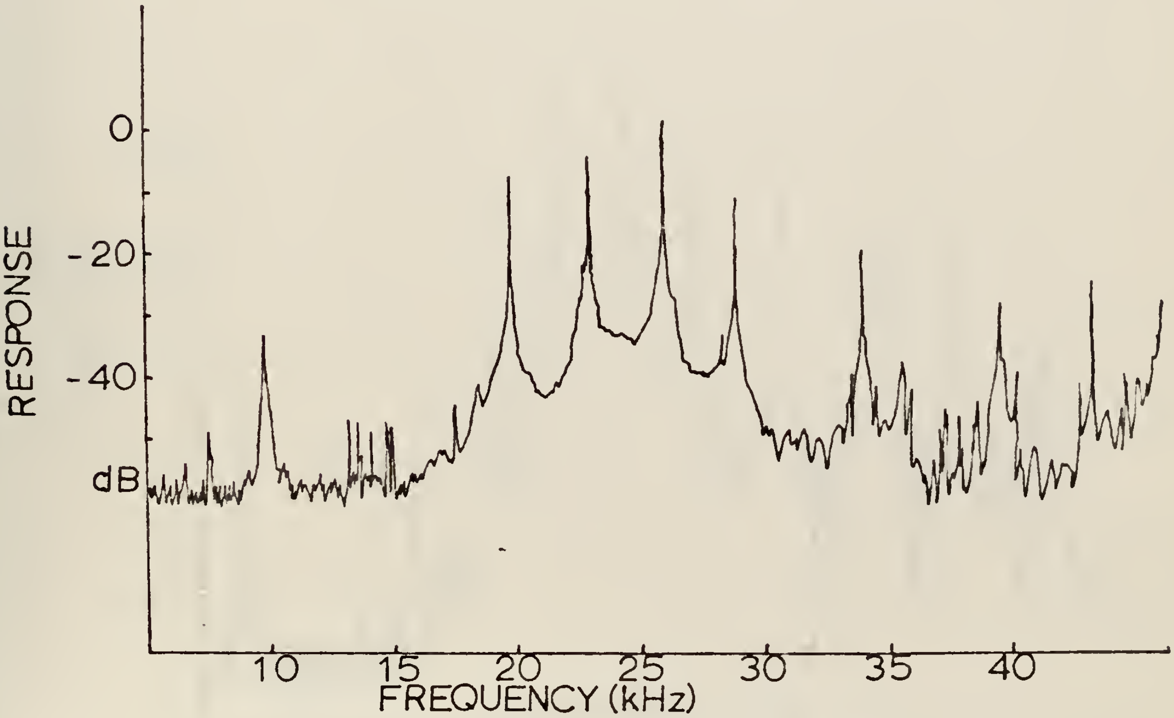

FIG. 6b OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR
with l = 11.22cm, D = 12.7cm, d = 5.1cm, t = 6mm

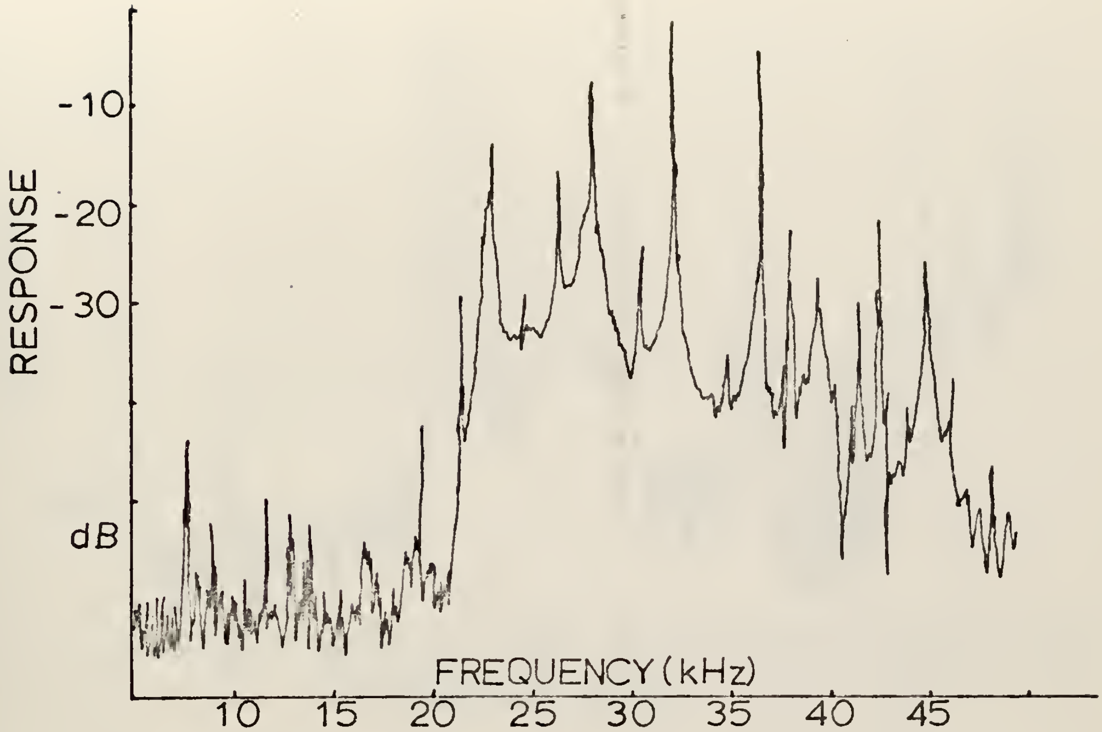

FIG. 6c OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR with l = 10cm, D = 12.7cm, d = 6.3 cm, t = 6mm

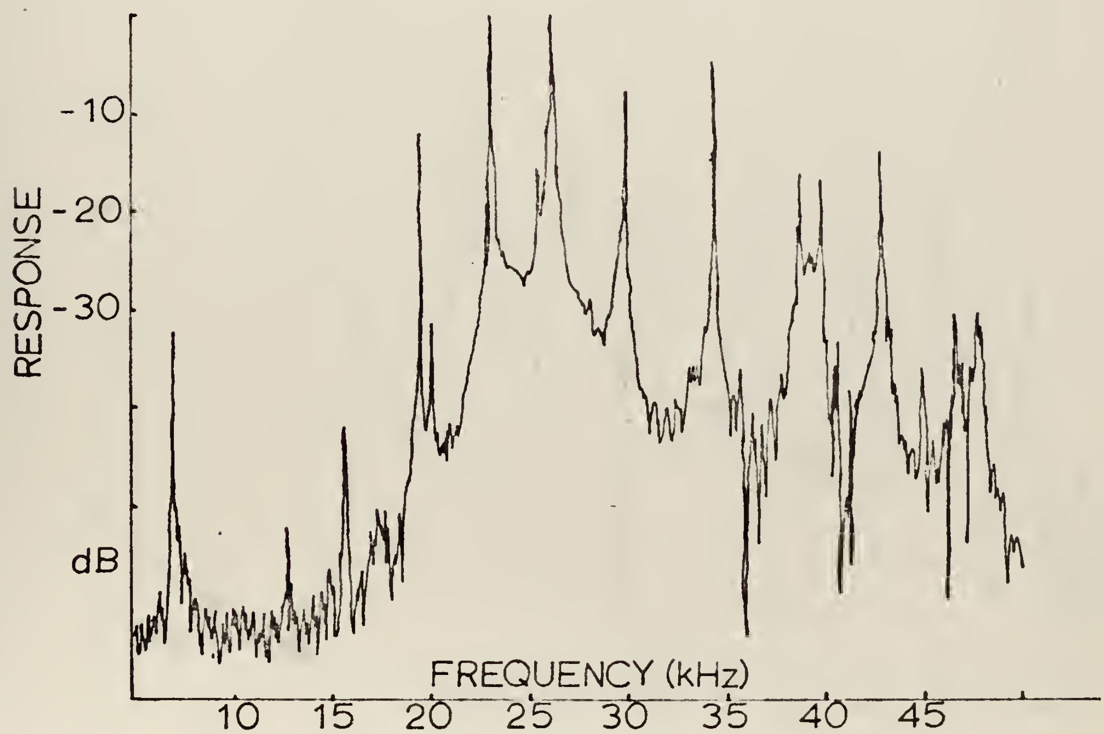

FIG. 6d OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR with l = 11.3cm, D = 12.7cm, d = 6.3cm, t = 6mm

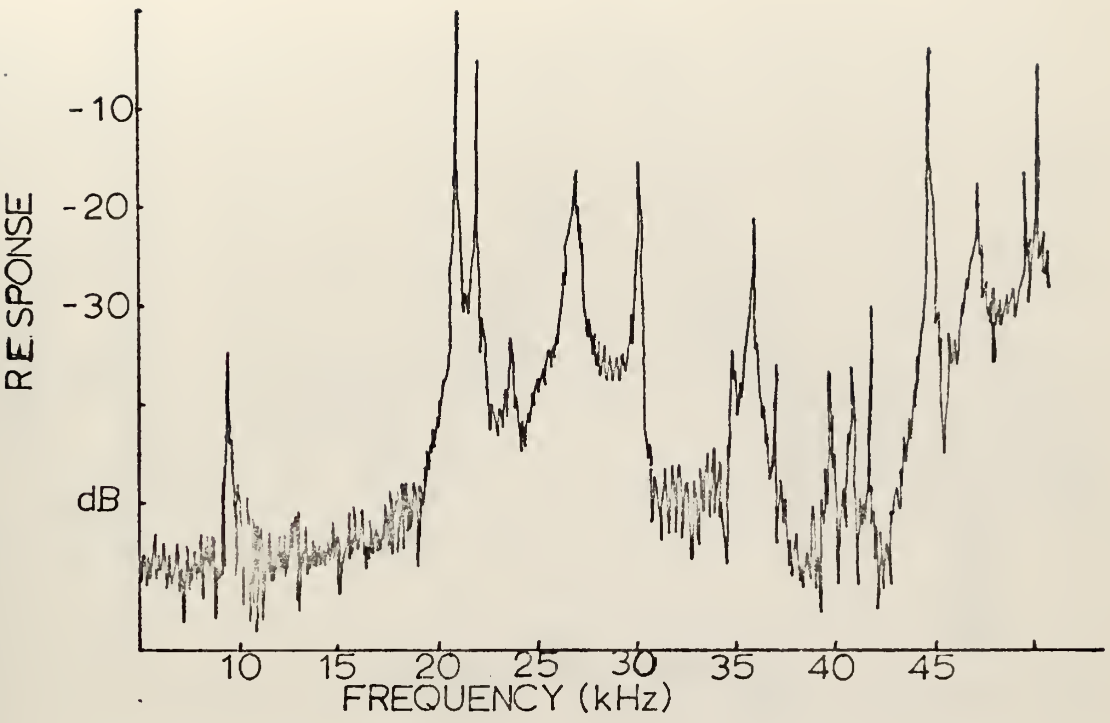

FIG. 6e OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR
with l = 10.0cm, D = 12.7cm, d = 5.1cm, t = 6mm

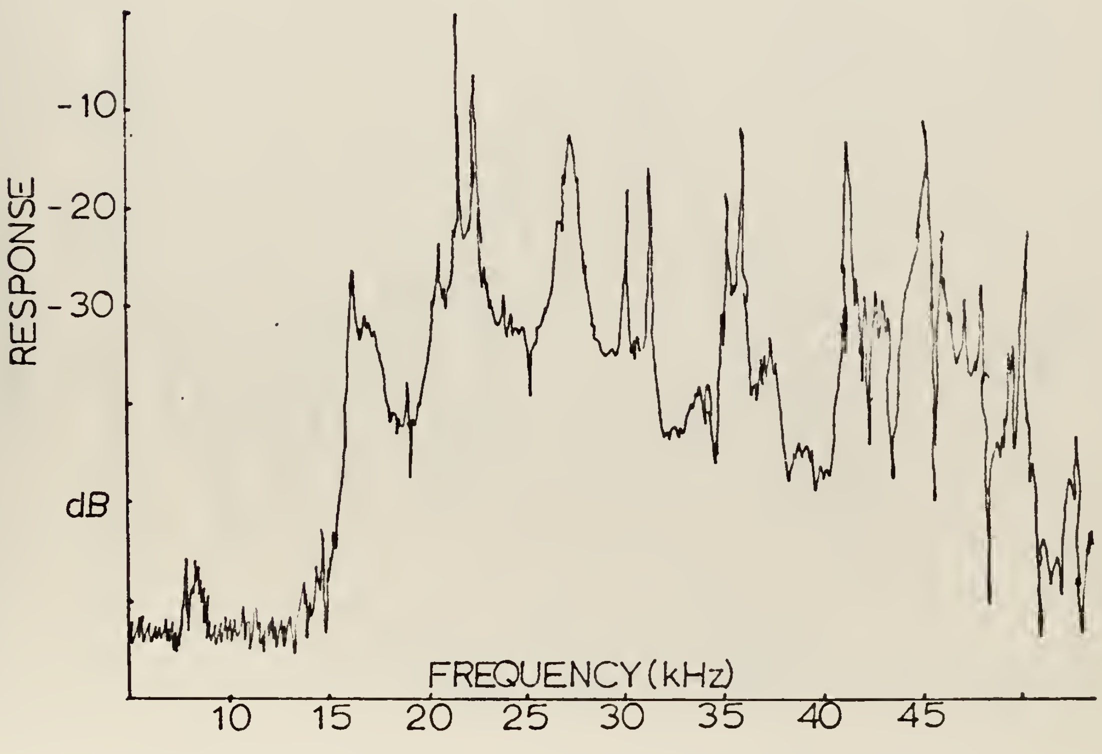

FIG. 6f OUTPUT RESPONSE VS FREQUENCY FOR RESONATOR
with l = 10.0cm, D = 12.7cm, d = 6.3 cm, t = 3mm

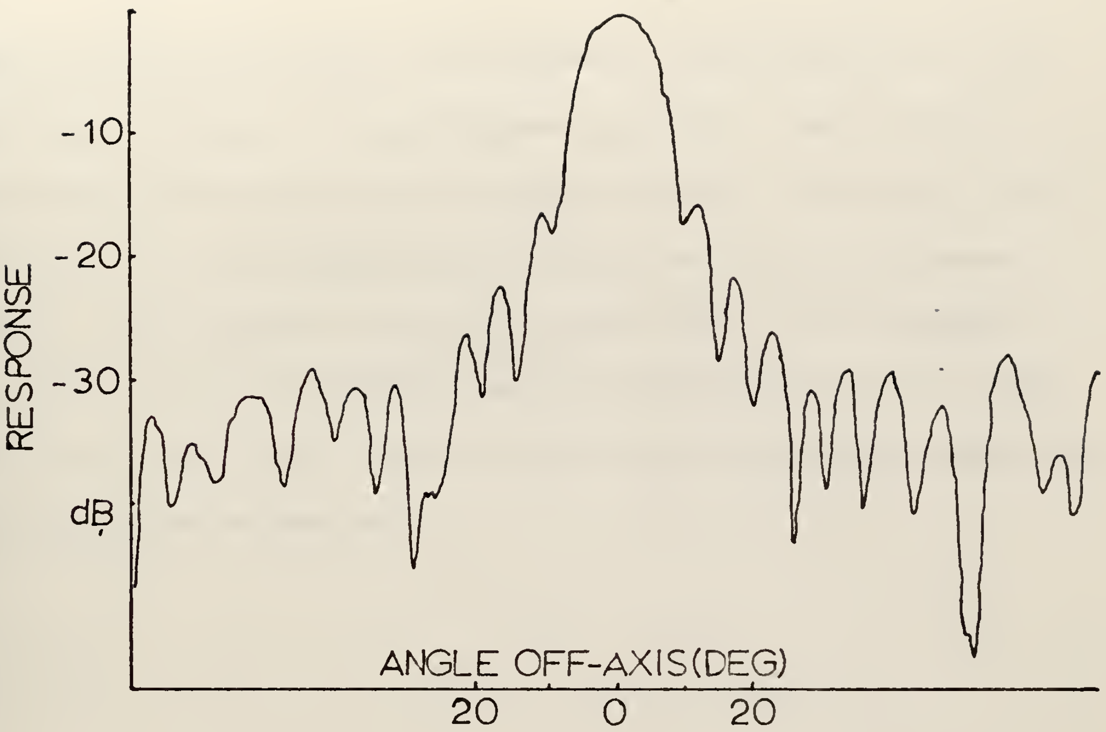

FIG. 7 DIRECTIVITY PATTERN FOR A LONGITUDINAL MODE RESONANCE

FIG. 8 DIRECTIVITY PATTERN FOR A DOUBLET MODE RESONANCE

appears for the $\lambda/2$ and the $2\ \lambda/2$ resonance. The directivity of the doublet mode resonance is characterized by a triple lobe structure, a strong center lobe of about five degrees beam width and a minor lobe on either side 2 dB-10 dB down from the center lobe (Fig. 8). Figures 7 and 8 are directivities of the same resonator (Fig. 6d) at 30,068Hz (longitudinal mode) and 26,259Hz (doublet) respectively. This triple lobe structure was found to occur for all modes investigated except the longitudinal mode resonances.

V. SPEAKERS AS SOURCES

Both of the general methods described in Section II were used with speakers as the acoustic source of the primaries. A variation of method 1, utilizing a horn arrangement (Fig. 9) to bring the two sound fields together seems to be a promising technique. Some evidence of difference-frequency generation was noted. However, sound pressure levels for the primaries of 118 dB re .0002 μbar at the last maximum in the near field were the highest measured and this is probably not sufficient for development of a measurable parametric difference-frequency.

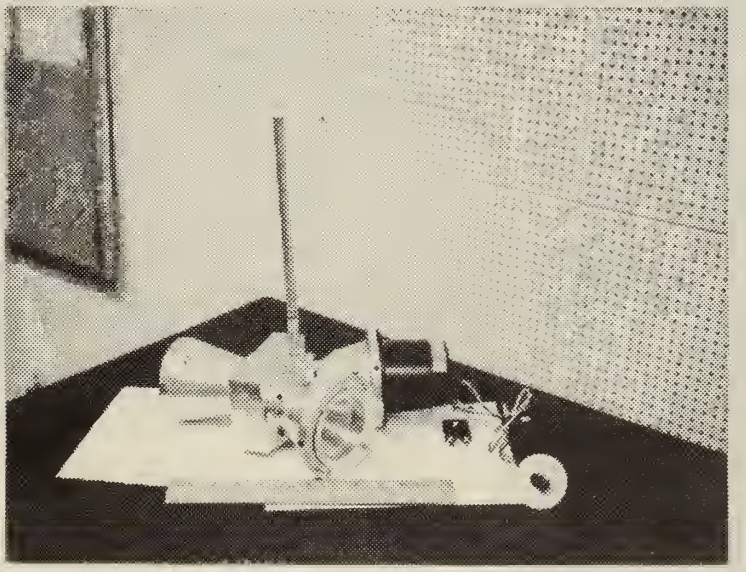

FIG. 9 HORN ARRANGEMENT FOR USE WITH TWO SPEAKERS

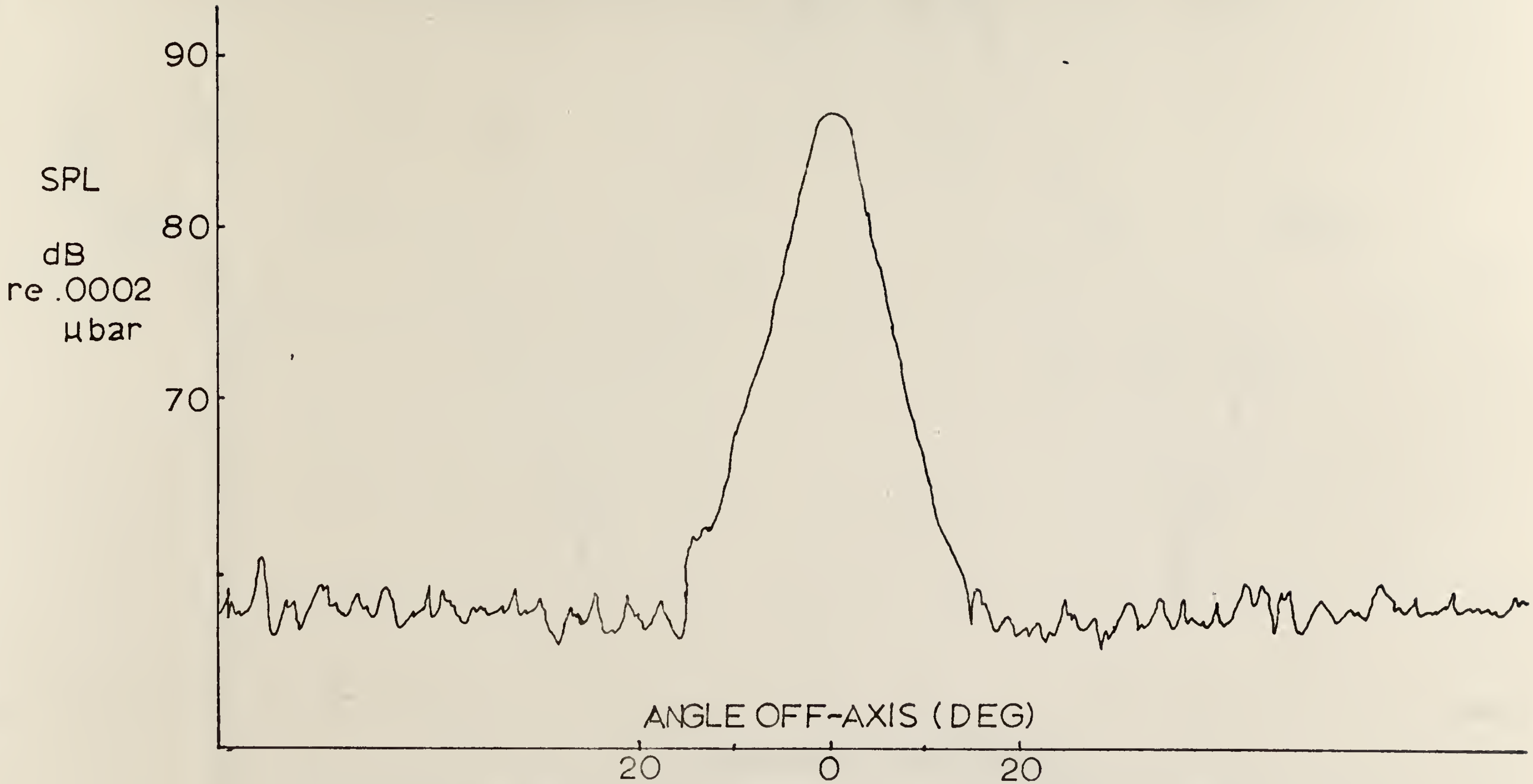

FIG. 10 DIRECTIVITY PATTERN OF PARAMETRIC DIFFERENCE-FREQUENCY (9.4kHz) 40cm FROM SOURCE

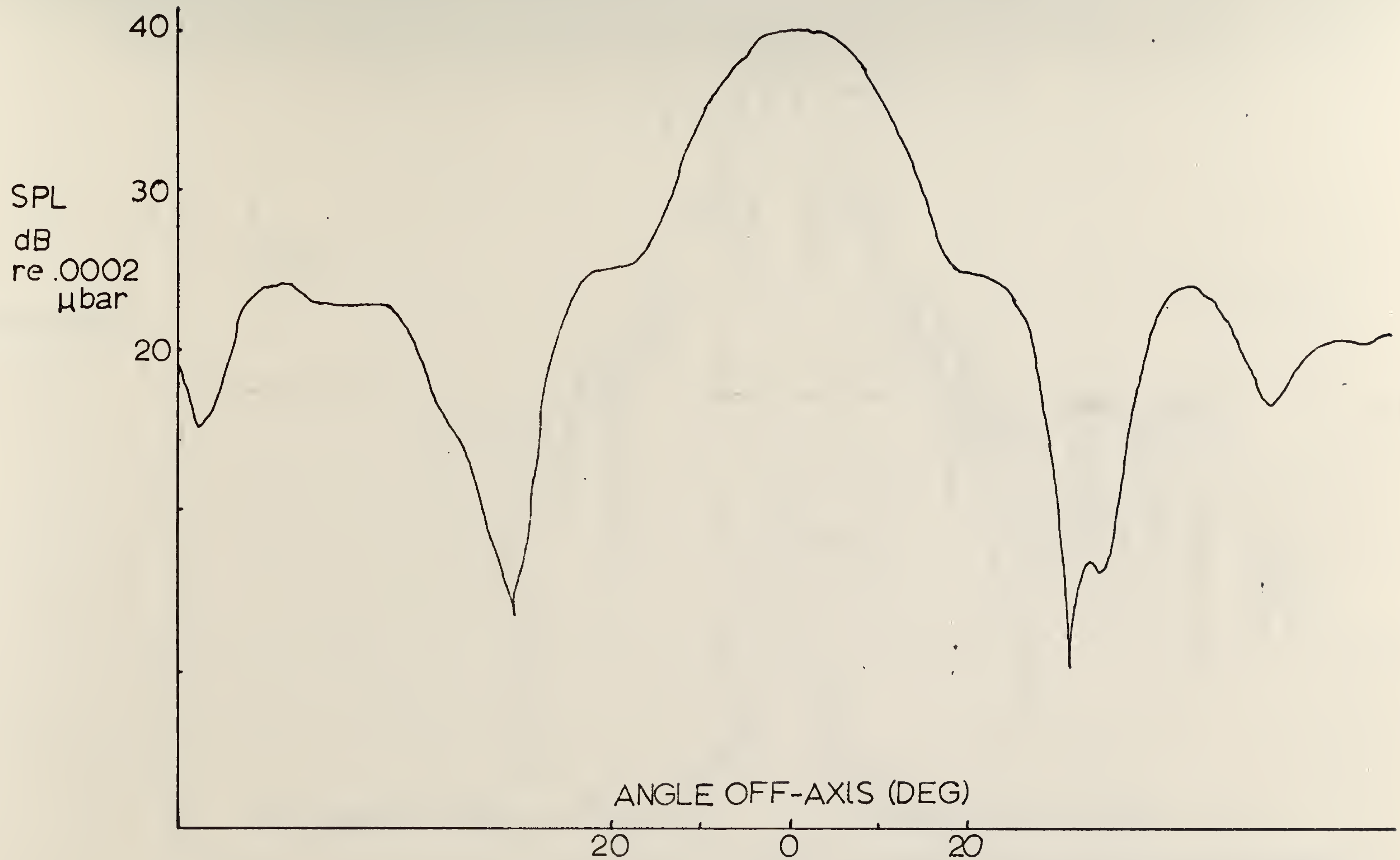

FIG. 11 DIRECTIVITY PATTERN OF DIRECT-DRIVE 9.4kHz SIGNAL 40cm FROM SOURCE

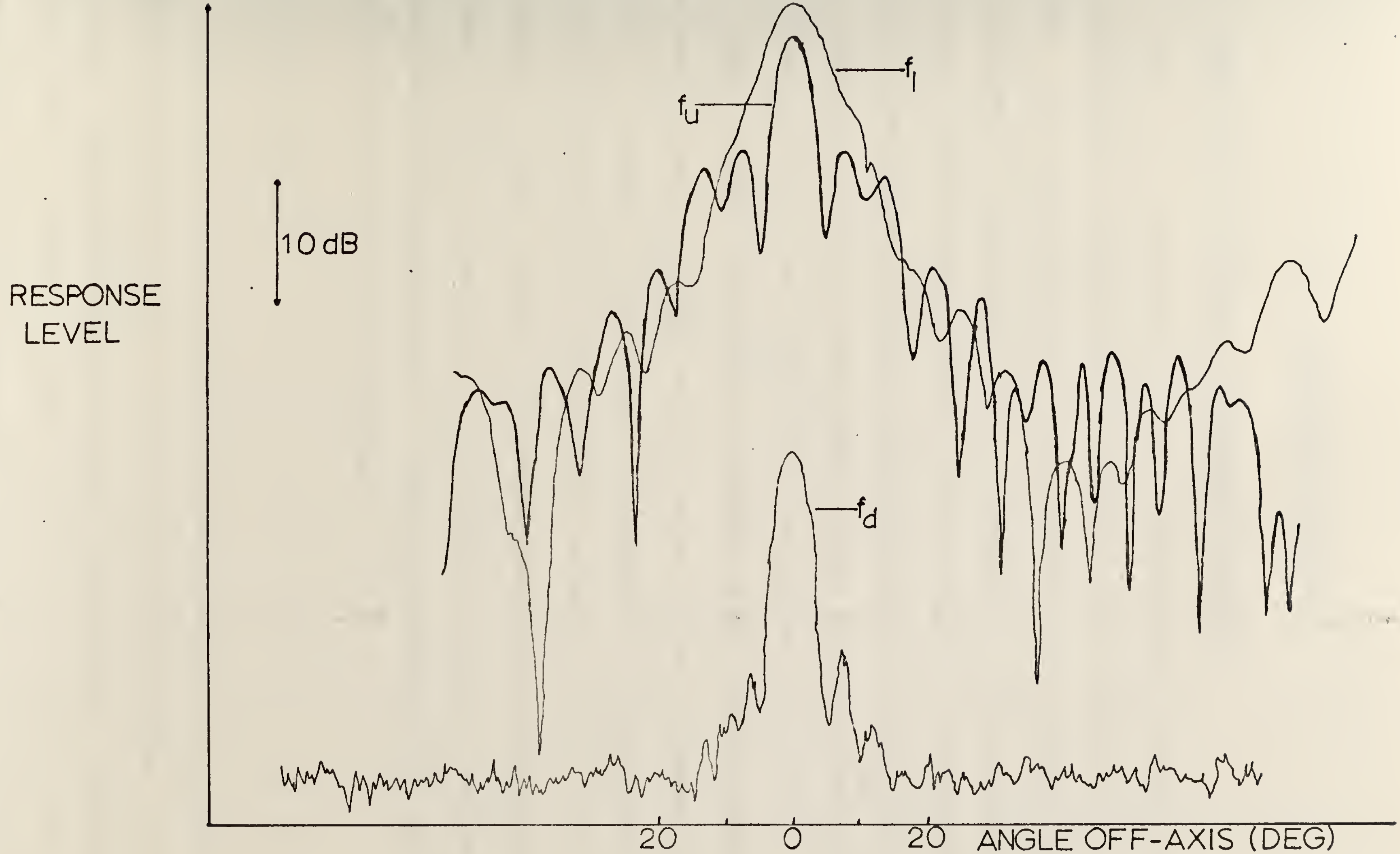

FIG. 12 DIRECTIVITY PATTERNS FOR THE LOWER AND UPPER PRIMARY FREQUENCIES AND THE PARAMETRIC DIFFERENCE-FREQUENCY GENERATED BY THEM

VI. EXPERIMENTAL RESULTS

A. DIRECTIVITY AND STRENGTH OF DIFFERENCE-FREQUENCY

Figure 10 is a directivity plot of a parametrically generated difference-frequency at 9.4kHz (f_d). The beamwidth is 5.4 degrees at the 3 dB point. The primary frequencies used to generate this difference frequency were 20.0kHz (f_l) and 29.4kHz (f_u) the longitudinal mode resonances of the same resonator whose response is shown in Fig. 6a. The source levels for the upper and lower primary frequencies were 128 dB and 121 dB re .0002 μbar. The source level (SPLat 1 meter from the source) of the difference-frequency was 73 dB. Driving voltage was sixty volts.

Figure 11 shows a directivity plot of the same resonator driven at an input frequency of 9.4kHz and sixty volts. It may be compared to Fig. 10 to contrast the beam width, 14.4 degrees at the half-power point, of a direct-drive signal versus that of a parametric signal at the same frequency. Figure 12 shows the directivity patterns for the primary frequencies with the pattern for the difference-frequency superimposed. The difference-frequency appears to take on the characteristic directivity of the most directive of the primaries.

B. SPL VS DISTANCE

Measurements of the on-axis sound pressure levels for all three frequencies were made at various distances from the source (Fig. 13) in an attempt to show difference-frequency growth with distance. Because of difficulties in assuring an on-axis measurement for such highly directive sound beams, several data points may be three to five decibels lower

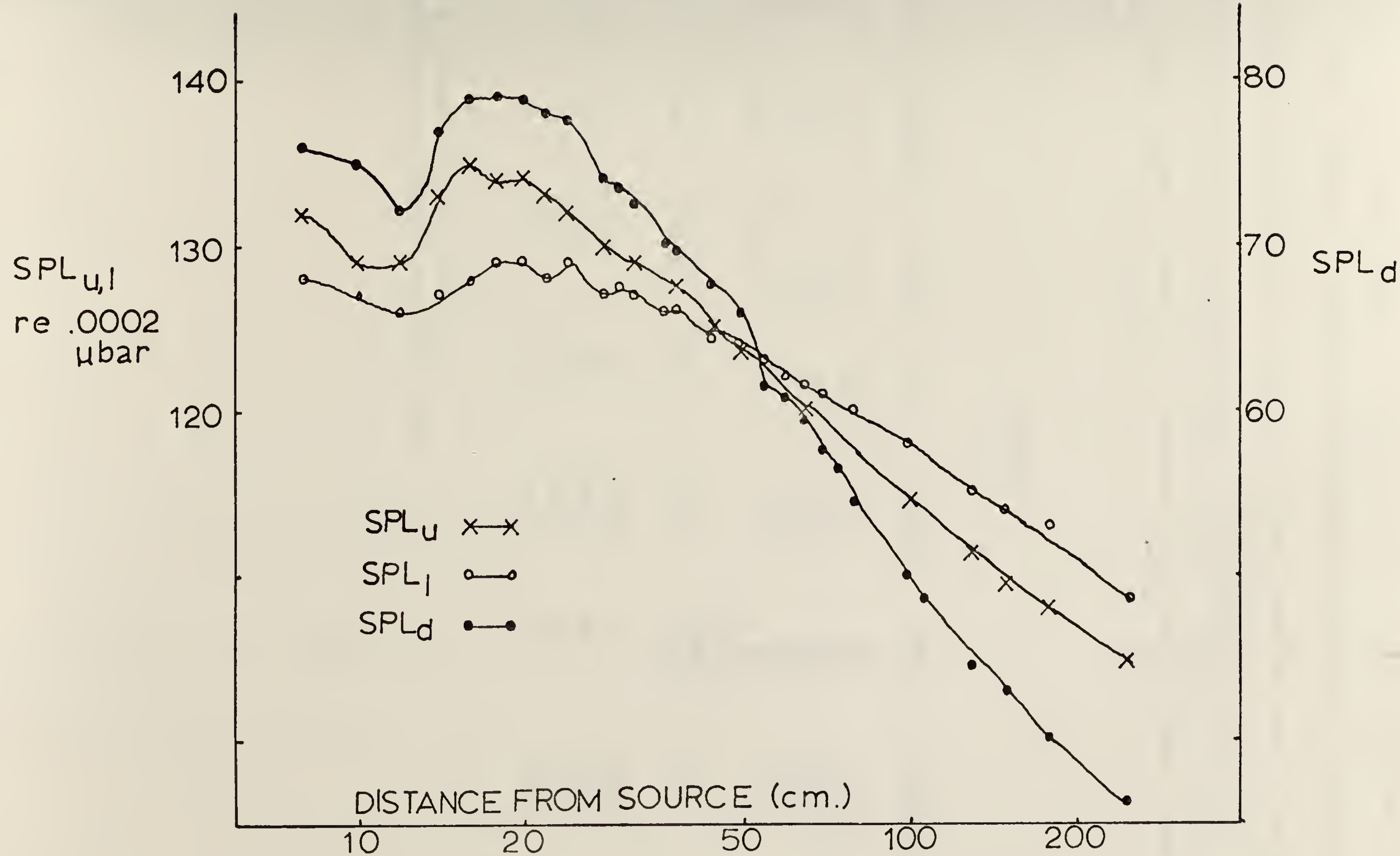

FIG. 13 SOUND PRESSURE LEVELS FOR THE LOWER AND UPPER PRIMARIES AND THE DIFFERENCE-FREQUENCY VERSUS DISTANCE FROM SOURCE

than the actual SPL. Though the data points in Fig. 13 indicate a greater than 20 dB per decade rate-of-decay for the difference-frequency in the far field, numerous subsequent measurements using a more careful alignment procedure from 50 cm to 3.7 meters distance from the source indicate a very nearly 20 dB per decade decay for the difference-frequency.

Table 1 gives a summary of pertinent data for different experimental runs.

TABLE 1

Run	f_d (kHz)	SPL_d	B W	f_u (kHz)	SPL_u	f_u mode
A	3.0	41	6.5	26.1	121	doublet
B	6.2	49	6.3	26.1	116	doublet
C	9.1	50		29.1	118	long.
D	9.4	73	5.4	29.4	128	long.

Run	f_1 (kHz)	SPL_1	f_1 mode	Driving voltage (RMS)	NCP
A	23.1	124	doublet	60	-204
B	19.8	118	long.	80	-185
C	20.0	115	long.	60	-183
D	20.0	121	long.	60	-176

B W - Beamwidth in degrees
SPL's at one meter (re. .0002 ubar)

VII. DISCUSSION OF RESULTS

Theory predicts that a parametrically derived difference-frequency will show an increased directivity over a direct-drive signal from the same source at the same frequency. This directivity arises from the length of the zone of interaction of the primaries. In this respect the parametric generator can be thought of as an end-fire array. Comparison of Figs. 10 and 11 dramatically illustrate the increased directivity of the parametric signal. The absence of side-lobes in Fig. 10 is another significant point that should be noted.

Table 1 introduces a nonlinear conversion parameter (NCP). This parameter is defined as SPL_d-SPL_u-SPL_1 and is a convenient means of measuring the coupling of the primaries into the difference-frequency. The values of NCP in Table 1 indicate an almost square-law dependence on frequency (Fig. 14).

In one experiment, four different types of calibrated microphones were used to measure the difference-frequency at the same distance from the source. Each microphone gave an identical sound pressure level measurement. This was done to eliminate the possibility that some nonlinear effect might be occurring in the microphone itself, thereby giving an erroneous measurement. It is argued that if there were such an effect it would be different for microphones of different sizes and types. The microphones used were all calibrated by a comparison method using the Bruel and Kjaer microphone described in section II as the standard.

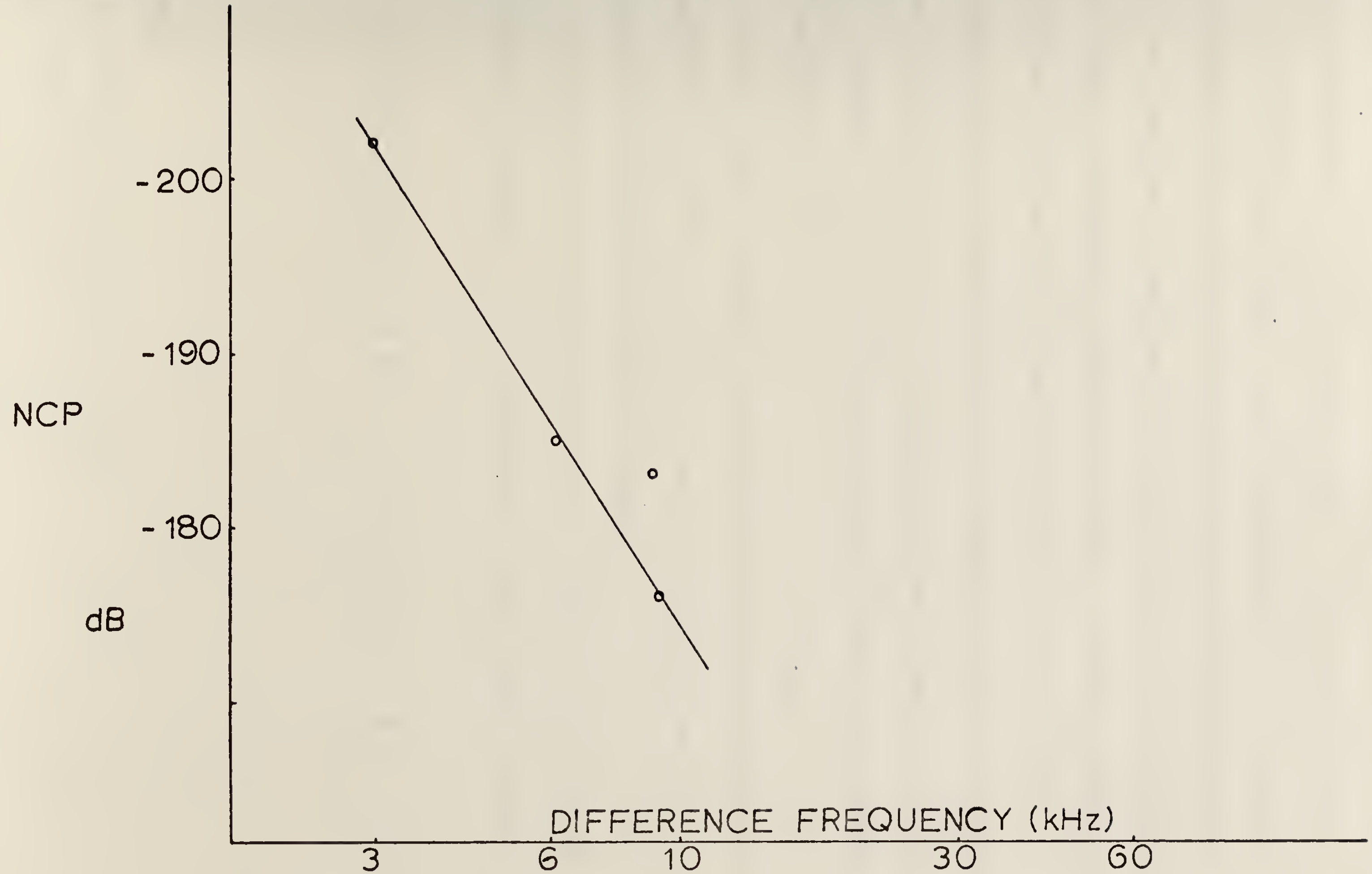

FIG. 14 NONLINEAR CONVERSION PARAMETER

Besides the Bruel and Kjaer the other microphones used were: an Electrovoice omnidirectional "Electret" model 1171 with a sensitivity of -71 dB below 15kHz; an Altec model 21 BR-150 with a microphone capsule of one-half inch diameter and sensitivity of -55 dB below 10kHz. All sensitivities re 1 volt/.0002 μbar.

In another experiment, a reflector was placed in the sound field at a position fifty centimeters from the face of the resonator. The reflector was designed to redirect the shorter wavelength primaries and to pass the longer wavelength difference-frequency. On-axis measurements were then made with and without the reflector in the sound field. The sound pressure level of the primaries decreased by thirty-six decibels while the SPL of the difference-frequency only decreased by twenty-three decibels. This suggests that the difference-frequency does exist independent of the primaries.

There is no doubt that the difference-frequency existed during the experiments conducted for this study. It could be heard by the human ear and its directivity could be sensed.

Muir [Ref. 5] derived a formula for computing the pressure of the difference-frequency. For comparison to experimental results it is shown:

$$p_d = \frac{\beta \omega_d^2 \Phi_e^2 r_o^2 \, p_1 p_2}{32 \rho_o c_o^4 R_o} \left[\left(\frac{\alpha_1 + \alpha_2}{2} \right)^2 + k_d^2 \sin^4 \left(\frac{\theta}{2} \right) \right]^{-1/2}$$

where

β = constant of nonlinearity $\left(\frac{\gamma + 1}{2}\right)$

p_d = difference-frequency pressure

ω_d = angular difference-frequency

ϕ_e = half-power beamwidth of zone of interaction

r_o = range at which spherical divergence begins

p_1, p_2 = average peak wave pressures in near-field of primaries

R_o = distance from origin to field point

α_n = small signal attenuation coefficient

θ = off-axis angle

The values used for a computation of p_d were:

$\beta = 1.201$

ϕ_e = .44 radians - the approximate half-power beam width of the primaries, run D.

r_o = 25 cm

$p_1, p_2 = 63.2\ \text{Nt/m}^2$ (130 dB re .0002 μbar)

$$\frac{\alpha_1 + \alpha_2}{2} = 1.26 \times 10^{-2}$$

$R_o = 1$

$\theta = 0$ (on axis)

Applying the appropriate values from run D of Table I, a theoretical sound pressure level of 103 dB re .0002 μbar at one meter from the source can be calculated. A sound pressure level of 73 dB was measured experimentally.

VIII. CONCLUSIONS

Parametric amplification of sound in air does generate a difference-frequency component. This difference-frequency component demonstrates a direct dependence on the frequency difference of the primaries and the strength of the primaries. The directivity of the difference-frequency assumes a character very near the directivity of the most directive primary.

The St. Clair resonator is a convenient sound source for producing high-intensity signals with the characteristics necessary to generate a parametric signal. However, the high-Q of its resonant frequencies limit it applications.

LIST OF REFERENCES

1. St. Clair, H. W., "An Electromagnetic Sound Generator for Producing Intense High Frequency Sound," Rev. Sci. Instr., v. 12, p. 250-256, May 1941.

2. Applied Research Laboratories, University of Texas, Report ARL-TR-71-33, The Interaction of Two Plane Sound Waves of Finite Amplitude and Coincident Direction of Propagation by K. Brinkmann, translation by P. J. Welton, 1 October 1972.

3. Bancroft, D., "The Velocity of Longitudinal Waves in Cylindrical Bars," Phys. Rev., v. 59, p. 588-593, 1 April 1941.

4. Booker, R. E. and Sagar, F. H., "Velocity Dispersion of the Lowest-Order Longitudinal Mode in Finite Rods of Circular Cross Section," The Journal of the Acoustical Society of America, v. 49, p. 1491-1498, May 1971.

5. Applied Research Laboratories, University of Texas, Report ARL-TR-71-1, An Analysis of the Parametric Acoustic Arrays for Spherical Wave Fields, by T. G. Muir, p. 29, May 1971.

DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Asst Professor A. I. Eller, Code 61 Er Department of Physics and Chemistry Naval Postgraduate School Monterey, California 93940	4
4.	LT W. P. Shealy, USN USS Cayuga (LST-1186) FPO San Francisco, California 96601	1

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Difference-frequency						
Parametric amplification						
ST. CLAIR resonator						
Difference-frequency generation in air						
Combination waves						
Nonlinear interaction						
Directivity patterns						
Longitudinal mode resonance						
Doublet mode resonance						
High intensity source						
Nonlinear conversion parameter						

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassfied
	2b. GROUP

REPORT TITLE

Parametric Difference-Frequency Generation of Sound in Air

. DESCRIPTIVE NOTES (Type of report and, inclusive dates)
Master's Thesis; December, 1972

. AUTHOR(S) (First name, middle initial, last name)
William P. Shealy

. REPORT DATE December, 1972	7a. TOTAL NO. OF PAGES 34	7b. NO. OF REFS 5

a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California

13. ABSTRACT

This study explores the feasibility of parametric difference-frequency generation in air through the interaction of two intense high-frequency sound waves. The sound pressure levels at various distances from the source are shown and the directivity of the difference-frequency is described.

An experimental apparatus utilizing a modified St. CLAIR resonator to generate the high-intensity primary signals is explained and some characteristics of the resonator are described.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

Thesis 143214
S43745 Shealy
c.1
Parametric difference-
frequency generation of
sound in air.